Hayter?

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1.1.2 $S = \{0 \text{ females}, 1 \text{ female}, 2 \text{ females}, 3 \text{ females}, \ldots, n \text{ females}\}$

1.1.6 $S = \{(\text{red, shiny}), (\text{red, dull}), (\text{blue, shiny}), (\text{blue, dull})\}$

1.2.6 In Figure 1.10, let (x, y) represent the outcome that the score on the red die is x and the score on the blue die is y. The event that the score on the red die is *strictly greater* than the score on the blue die consists of the following 15 outcomes:

{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4), (6,1), (6,2), (6,3), (6,4), (6,5)}

The probability of each outcome is $\frac{1}{36}$, so the required probability is $15 \times \frac{1}{36} = \frac{5}{12}$. This probability is less than 0.5 because of the possibility that both scores are equal. The complement of this event is the event that the red die has a score *less than or equal* to the score on the blue die with a probability of $1 - \frac{5}{12} = \frac{7}{12}$.

1.2.10 (a) See Figure 1.24.
P(Type I battery lasts longest) = P((II, III, I)) + P((III, II, I))
= 0.39 + 0.03 = 0.42.

(b) P(Type I battery lasts shortest) = P((I, II, III)) + P((I, III, II))
= 0.11 + 0.07 = 0.18.

(c) P(Type I battery does not last longest) = 1 − P(Type I battery lasts longest)
= 1 − 0.42 = 0.58.

(d) P(Type I battery last longer than Type II)
= P((II, I, III)) + P((II, III, I)) + P((III, II, I))
= 0.24 + 0.39 + 0.03 = 0.66.

1.3.8 $S = \{1, 2, 3, 4, 5, 6\}$ where each outcome is equally likely with a probability of $\frac{1}{6}$. The events A, B, and B' are A = {2, 4, 6}, B = {1, 2, 3, 5} and $B' = \{4, 6\}$.

(a) $A \cap B = \{2\}$ so $P(A \cap B) = \frac{1}{6}$.
(b) $A \cup B = \{1, 2, 3, 4, 5, 6\}$ so $P(A \cup B) = 1$.
(c) $A \cap B' = \{4, 6\}$ so $P(A \cap B') = \frac{2}{6} = \frac{1}{3}$.

1.3.12 Let the event R be that a red ball is chosen and let the event S be that a shiny ball is chosen.
It is known that $P(R \cap S) = \frac{55}{200}$, $P(S) = \frac{91}{200}$ and $P(R) = \frac{79}{200}$.
Hence the probability that the chosen ball is either shiny or red is

$$P(R \cup S) = P(R) + P(S) - P(R \cap S) = \frac{79}{200} + \frac{91}{200} - \frac{55}{200} = \frac{115}{200} = 0.575.$$

The probability of a dull blue ball is $P(R' \cap S') = 1 - P(R \cup S)$
$= 1 - 0.575 = 0.425$.

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1.4.6 Let the event O be an on time repair and let the event S be a satisfactory repair. It is known that $P(S \mid O) = 0.85$ and $P(O) = 0.77$.
The question asks for $P(O \cap S)$ which is
$P(O \cap S) = P(S \mid O) \times P(O) = 0.85 \times 0.77 = 0.6545.$

1.4.10 See Figure 1.25 in text.

(a) Let A be the event *both lines at full capacity* consisting of the outcome {(F,F)}.
Let B be the event *neither line is shut down* consisting of the outcomes {(P,P), (P,F), (F,P), (F,F)}.
Therefore $A \cap B$ = {(F,F)} and hence

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.19}{(0.14+0.2+0.21+0.19)} = 0.257.$$

(b) Let C be the event *at least one line at full capacity* consisting of the outcomes {(F,P), (F,S), (F,F), (S,F), (P,F)}.
Then $C \cap B$ = {(F,P), (F, F), (P,F)} and hence

$$P(C \mid B) = \frac{P(C \cap B)}{P(B)} = \frac{0.21 + 0.19 + 0.2}{0.74} = 0.811.$$

(c) Let D be the event that *one line is at full capacity* consisting of the outcomes {(F,P), (F,S), (P,F), (S,F)}.
Let E be the event *one line is shut down* consisting of the outcomes {(S,P), (S,F), (P,S), (F,S)}.
Then $D \cap E$ = {(F,S), (S,F)} and hence

$$P(D \mid E) = \frac{P(D \cap E)}{P(E)} = \frac{0.06+0.05}{0.06+0.05+0.07+0.06} = 0.458.$$

(d) Let G be the event that *neither line is at full capacity* consisting of the outcomes {(S,S), (S,P), (P,S), (P,P)}.
Let H be the event that *at least one line is at partial capacity* consisting of the outcomes {(S,P), (P,S), (P,P), (P,F), (F,P)}.
Then $F \cap G$ = {(S,P), (P,S), (P,P)} and hence

$$P(F \mid G) = \frac{P(F \cap G)}{P(G)} = \frac{0.06 + 0.07 + 0.14}{0.06 + 0.07 + 0.14 + 0.2 + 0.21} = 0.397.$$

1.5.10 P(no broken bulbs) $= \frac{83}{100} \times \frac{83}{100} \times \frac{83}{100} = 0.5718$

P(one broken bulb) = P(broken, not broken, not broken)
+ P(not broken, broken, not broken) +P(not broken, not broken, broken)
$= \left(\frac{17}{100} \times \frac{83}{100} \times \frac{83}{100}\right) + \left(\frac{83}{100} \times \frac{17}{100} \times \frac{83}{100}\right) + \left(\frac{83}{100} \times \frac{83}{100} \times \frac{17}{100}\right) = 0.3513.$

P(no more than one broken bulb in the sample)
= P(no broken bulbs) + P(one broken bulb) = 0.5718 + 0.3513 = 0.9231.

The probability of finding no broken bulbs increases with replacement, but the probability of finding no more than one broken bulb decreases with replacement.

1.6.4 The following information is given: P(Species 1) = 0.45, P(Species 2) = 0.38, P(Species 3) = 0.17, P(Tagged | Species 1) = 0.10, P(Tagged | Species 2) = 0.15, P(Tagged | Species 3) = 0.50.

Therefore, P(Tagged) = (P(Tagged | Species 1) × P(Species 1))
+ (P(Tagged | Species 2) × P(Species 2)) + (P(Tagged | Species 3) × P(Species 3))
= (0.10 × 0.45) + (0.15 × 0.38) + (0.50 × 0.17) = 0.187.

$$P(\text{Species 1} \mid \text{Tagged}) = \frac{P(\text{Tagged} \cap \text{Species 1})}{P(\text{Tagged})}$$
$$= \frac{P(\text{Species 1}) \times P(\text{Tagged} \mid \text{Species 1})}{P(\text{Tagged})} = \frac{0.45 \times 0.10}{0.187} = 0.2406.$$

$$P(\text{Species 2} \mid \text{Tagged}) = \frac{P(\text{Tagged} \cap \text{Species 2})}{P(\text{Tagged})}$$
$$= \frac{P(\text{Species 2}) \times P(\text{Tagged} \mid \text{Species 2})}{P(\text{Tagged})} = \frac{0.38 \times 0.15}{0.187} = 0.3048.$$

$$P(\text{Species 3} \mid \text{Tagged}) = \frac{P(\text{Tagged} \cap \text{Species 3})}{P(\text{Tagged})}$$
$$= \frac{P(\text{Species 3}) \times P(\text{Tagged} \mid \text{Species 3})}{P(\text{Tagged})} = \frac{0.17 \times 0.50}{0.187} = 0.4545.$$

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1.7.4 Number of full meals $= 5 \times 3 \times 7 \times 6 \times 8 = 5{,}040$.
Number of meals with just soup or appetizer $= (5+3) \times 7 \times 6 \times 8 = 2{,}688$.

1.7.6 (a) Define the notation (2,3,1,4) to represent the result that the player who finished 1st in tournament 1 finished 2nd in tournament 2, the player who finished 2nd in tournament 1 finished 3rd in tournament 2, etc. The result (1,2,3,4) then indicates that each competitor received the same ranking in both tournaments. Altogether there are $4! = 24$ different results, each equally likely, and so this single result has a probability of $\frac{1}{24}$.

(b) The results where no player receives the same ranking in the two tournaments are (2,1,4,3), (2,3,4,1), (2,4,1,3), (3,1,4,2), (3,4,1,2) (3,4,2,1), (4,1,2,3), (4,3,1,2) and (4,3,2,1). There are nine of these and so the required probability is $\frac{9}{24} = \frac{3}{8}$.

2.1.4 (a)

x_i	0	1	2
p_i	0.5625	0.3750	0.0625

(b)

x_i	0	1	2
$F(x_i)$	0.5625	0.9375	1.000

(c) $x = 0$ is the most likely value.

2.2.8 (a)

$$\int_0^{10} A\,(e^{10-\theta} - 1)\, d\theta = 1 \quad \Rightarrow \quad A = (e^{10} - 11)^{-1} = 4.54 \times 10^{-5}.$$

(b)

$$F(\theta) = \int_0^{\theta} f(y)dy = \frac{e^{10} - \theta - e^{10-\theta}}{e^{10} - 11} \quad \text{for } 0 \le \theta \le 10.$$

(c) $1 - F(8) = 0.0002$.

2.3.7 P(3 sixes are rolled) $= \frac{1}{6} \times \frac{1}{6} \times \frac{1}{6} = \frac{1}{216}$.

Therefore E(net winnings) $= (-\$1 \times \frac{215}{216}) + (\$499 \times \frac{1}{216}) = \$1.31$.

If you can play the game a large number of times then you should play the game as often as you can.

2.3.8 The expected net winnings will be negative.

2.3.9

x_i	0	1	2	3	4	5
p_i	0.1680	0.2816	0.2304	0.1664	0.1024	0.0512

E(payment) $= (0 \times 0.1680) + (1 \times 0.2816) + (2 \times 0.2304) + (3 \times 0.1664) + (4 \times 0.1024) + (5 \times 0.0512) = 1.9072$

E(winnings) $= \$2 - \$1.91 = \$0.09$.

The expected winnings increase to 9 cents per game. Increasing the probability of scoring a three reduces the expected value of the difference in the scores of the two dice.

2.3.10 (a) $$E(X) = \int_4^6 x \frac{1}{x \ln(1.5)}\, dx = 4.94.$$

(b) Solving $F(x) = 0.5$ gives $x = 4.90$.

2.3.11 (a) $$E(X) = \int_0^4 x \frac{x}{8}\, dx = 2.67.$$

(b) Solving $F(x) = 0.5$ gives $x = \sqrt{8} = 2.83$.

2.3.12 $$E(X) = \int_{0.125}^{0.5} x\, 5.5054(0.5 - (x - 0.25)^2)\, dx = 0.3095.$$

Solving $F(x) = 0.5$ gives $x = 0.3081$.

2.3.13 $$E(X) = \int_0^{10} \frac{\theta}{e^{10} - 11}(e^{10-\theta} - 1)\, d\theta = 0.9977.$$

Solving $F(\theta) = 0.5$ gives $\theta = 0.6927$.

(d) Interquartile range = 2.93 − 0.50 = 2.43.

2.4.10 Adding and subtracting two standard deviations from the mean value gives

$$P(60.4 \leq X \leq 89.6) \geq 0.75.$$

Adding and subtracting three standard deviations from the mean value gives

$$P(53.1 \leq X \leq 96.9) \geq 0.89.$$

2.4.11 The interval (109.55, 112.05) is $(\mu - 2.5c, \mu + 2.5c)$ so Chebyshev's inequality gives

$$P(109.55 \leq X \leq 112.05) \geq 1 - \frac{1}{2.5^2} = 0.84.$$

2.4.12 $E(X^2) = (3^2 \times \frac{1}{20}) + (4^2 \times \frac{3}{20}) + (5^2 \times \frac{6}{20}) + (6^2 \times \frac{10}{20}) = \frac{567}{20}$.

$Var(X) = E(X^2) - (E(X))^2 = \frac{567}{20} - (\frac{105}{20})^2 = \frac{63}{80}$.
The standard deviation is $\sqrt{63/80} = 0.887$.

2.4.13 (a) $E(X^2) = \int_{10}^{11} \frac{4x^3(130-x^2)}{819} dx = 108.61538$. Therefore $Var(X) = E(X^2) - (E(X))^2 = 108.61538 - 10.418234^2 = 0.0758$ and the standard deviation is $\sqrt{0.0758} = 0.275$.

(b) Solving $F(x) = 0.8$ gives the 80th percentile of the resistance as 10.69, and solving $F(x) = 0.1$ gives the 10th percentile of the resistance as 10.07.

(c) Solving $F(x) = 0.25$ gives $x = 4.43$.
Solving $F(x) = 0.75$ gives $x = 5.42$.

(d) Interquartile range $= 5.42 - 4.43 = 0.99$.

2.4.6 (a)
$$E(X^2) = \int_0^4 x^2 \left(\frac{x}{8}\right) dx = 8.$$

Then $E(X) = \frac{8}{3}$ so $Var(X) = 8 - \left(\frac{8}{3}\right)^2 = \frac{8}{9}$.

(b) $\sigma = \sqrt{\frac{8}{9}} = 0.94$.

(c) Solving $F(x) = 0.25$ gives $x = 2$.
Solving $F(x) = 0.75$ gives $x = \sqrt{12} = 3.46$.

(d) Interquartile range $= 3.46 - 2.00 = 1.46$.

2.4.7 (a)
$$E(X^2) = \int_{0.125}^{0.5} x^2\, 5.5054(0.5 - (x - 0.25)^2)\, dx = 0.1073.$$

Then $E(X) = 0.3095$ so $Var(X) = 0.1073 - 0.3095^2 = 0.0115$.

(b) $\sigma = \sqrt{0.0115} = 0.107$.

(c) Solving $F(x) = 0.25$ gives $x = 0.217$.
Solving $F(x) = 0.75$ gives $x = 0.401$.

(d) Interquartile range $= 0.401 - 0.217 = 0.184$.

2.4.8 (a)
$$E(X^2) = \int_0^{10} \frac{\theta^2}{e^{10} - 11}(e^{10-\theta} - 1)\, d\theta = 1.9803.$$

Then $E(X) = 0.9977$ so $Var(X) = 1.9803 - 0.9977^2 = 0.985$.

(b) $\sigma = \sqrt{0.985} = 0.992$.

(c) Solving $F(\theta) = 0.25$ gives $\theta = 0.288$.
Solving $F(\theta) = 0.75$ gives $\theta = 1.385$.

(d) Interquartile range $= 1.385 - 0.288 = 1.097$.

2.4.9 (a)
$$E(X^2) = \int_0^{50} \frac{375.3\, r^2}{(r+5)^4}\, dr = 18.80.$$

Then $E(X) = 2.44$ so $Var(X) = 18.80 - 2.44^2 = 12.8$.

(b) $\sigma = \sqrt{12.8} = 3.58$.

(c) Solving $F(r) = 0.25$ gives $r = 0.50$.
Solving $F(r) = 0.75$ gives $r = 2.93$.

2.5.7 (a)

X\Y	0	1	2	p_{i+}
0	4/16	4/16	1/16	9/16
1	4/16	2/16	0	6/16
2	1/16	0	0	1/16
p_{+j}	9/16	6/16	1/16	

(b) See table above.

(c) No they are not independent. For example $p_{22} \neq p_{2+} \times p_{+2}$.

(d)

$$E(X) = \left(0 \times \frac{9}{16}\right) + \left(1 \times \frac{6}{16}\right) + \left(2 \times \frac{1}{16}\right) = \frac{1}{2}.$$

$$E(X^2) = \left(0^2 \times \frac{9}{16}\right) + \left(1^2 \times \frac{6}{16}\right) + \left(2^2 \times \frac{1}{16}\right) = \frac{5}{8}.$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{5}{8} - \left(\frac{1}{2}\right)^2 = \frac{3}{8} = 0.3676.$$

The random variable Y has the same mean and variance.

(e)

$$E(XY) = 1 \times 1 \times p_{11} = \frac{1}{8}.$$

$$Cov(X,Y) = E(XY) - (E(X) \times E(Y)) = \frac{1}{8} - \left(\frac{1}{2} \times \frac{1}{2}\right) = -\frac{1}{8}.$$

(f)

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = -\frac{1}{3}.$$

(g)

$$P(Y = 0|X = 0) = \frac{p_{00}}{p_{0+}} = \frac{4}{9}.$$

$$P(Y = 1|X = 0) = \frac{p_{01}}{p_{0+}} = \frac{4}{9}.$$

$$P(Y = 2|X = 0) = \frac{p_{02}}{p_{0+}} = \frac{1}{9}.$$

$$P(Y = 0|X = 1) = \frac{p_{10}}{p_{1+}} = \frac{2}{3}.$$

$$P(Y = 1|X = 1) = \frac{p_{11}}{p_{1+}} = \frac{1}{3}.$$

$$P(Y = 2|X = 1) = \frac{p_{12}}{p_{1+}} = 0.$$

2.5.8 (a)

$$\int_{x=0}^{5} \int_{y=0}^{5} A\,(20 - x - 2y)\; dx\, dy = 1 \quad \Rightarrow \quad A = 0.0032.$$

(b)
$$P(1 \le X \le 2, 2 \le Y \le 3) = \int_{x=1}^{2}\int_{y=2}^{3} 0.0032\,(20 - x - 2y)\,dx\,dy$$
$$= 0.0432.$$

(c)
$$f_X(x) = \int_{y=0}^{5} 0.0032\,(20 - x - 2y)\,dy = 0.016\,(15 - x)$$
for $0 \le x \le 5$.

$$f_Y(y) = \int_{x=0}^{5} 0.0032\,(20 - x - 2y)\,dx = 0.008\,(35 - 4y)$$
for $0 \le y \le 5$.

(d) No they are not independent since $f(x,y) \neq f_X(x)f_Y(y)$.

(e)
$$E(X) = \int_0^5 x\,0.016\,(15 - x)\,dx = \frac{7}{3}.$$
$$E(X^2) = \int_0^5 x^2\,0.016\,(15 - x)\,dx = \frac{15}{2}.$$
$$Var(X) = E(X^2) - E(X)^2 = \frac{15}{2} - \left(\frac{7}{3}\right)^2 = \frac{37}{18}.$$

(f)
$$E(Y) = \int_0^5 y\,0.008\,(35 - 4y)\,dy = \frac{13}{6}.$$
$$E(Y^2) = \int_0^5 y^2\,0.008\,(35 - 4y)\,dy = \frac{20}{3}.$$
$$Var(Y) = E(Y^2) - E(Y)^2 = \frac{20}{3} - \left(\frac{13}{6}\right)^2 = \frac{71}{36}.$$

(g)
$$f_{Y|X=3}(y) = \frac{f(3,y)}{f_X(3)} = \frac{17 - 2y}{60} \quad \text{for} \quad 0 \le y \le 5.$$

(h)
$$E(XY) = \int_{x=0}^{5}\int_{y=0}^{5} 0.0032\,xy\,(20 - x - 2y)\,dx\,dy = 5.$$
$$Cov(X,Y) = E(XY) - (E(X) \times (EY)) = 5 - \left(\frac{7}{3} \times \frac{13}{6}\right) = -\frac{1}{18}.$$

(i)
$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = -0.0276$$

$$E(X^2) = \left(2^2 \times \frac{2}{10}\right) + \left(3^2 \times \frac{3}{10}\right) + \left(4^2 \times \frac{3}{10}\right) + \left(5^2 \times \frac{2}{10}\right) = \frac{133}{10}.$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{133}{10} - \left(\frac{7}{2}\right)^2 = \frac{21}{20}.$$

2.7.4 Let X_i be the value of the i^{th} card dealt. Then

$$E(X_i) = \left(2 \times \frac{1}{13}\right) + \left(3 \times \frac{1}{13}\right) + \left(4 \times \frac{1}{13}\right) + \left(5 \times \frac{1}{13}\right) + \left(6 \times \frac{1}{13}\right)$$

$$+ \left(7 \times \frac{1}{13}\right) + \left(8 \times \frac{1}{13}\right) + \left(9 \times \frac{1}{13}\right) + \left(10 \times \frac{1}{13}\right) + \left(15 \times \frac{4}{13}\right) = \frac{114}{13}.$$

The total score of the hand is $Y = X_1 + \ldots + X_{13}$ which has an expectation

$$E(Y) = E(X_1) + \ldots + E(X_{13}) = 13 \times \frac{114}{13} = 114.$$

2.7.5 (a) $$\int_1^{11} A \left(\frac{3}{2}\right)^x dx = 1 \Rightarrow A = \frac{\ln(1.5)}{1.5^{11} - 1.5} = \frac{1}{209.6}.$$

(b) $$F(x) = \int_1^x \frac{1}{209.6} \left(\frac{3}{2}\right)^y dy = 0.01177 \left(\frac{3}{2}\right)^x - 0.01765$$

for $1 \leq x \leq 11$.

(c) Solving $F(x) = 0.5$ gives $x = 9.332$.

(d) Solving $F(x) = 0.25$ gives $x = 7.706$.
Solving $F(x) = 0.75$ gives $x = 10.305$.
The interquartile range is $10.30 - 7.71 = 2.599$.

2.7.6 (a) $$f_X(x) = \int_1^2 4x(2-y)\, dy = 2x \quad \text{for} \quad 0 \leq x \leq 1.$$

(b) $$f_Y(y) = \int_0^1 4x(2-y)\, dx = 2(2-y) \quad \text{for} \quad 1 \leq y \leq 2.$$

Since $f(x,y) = f_X(x) \times f_Y(y)$ the random variables are independent.

(c) $Cov(X,Y) = 0$ since the random variables are independent.

(d) $f_{X|Y=1.5}(x) = f_X(x)$ since the random variables are independent.

$X \sim B(6, 0.7)$

x_i	0	1	2	3	4	5	6
p_i	0.0007	0.0102	0.0595	0.1852	0.3241	0.3025	0.1176

$E(X) = 6 \times 0.7 = 4.2$ and $Var(X) = 6 \times 0.7 \times 0.3 = 1.26$ with $\sigma = \sqrt{1.5} = 1.12$.

3.1.4 $X \sim B(9, 0.09)$.

(a) $P(X = 2) = 0.1507$.

(b) $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 0.1912$.

$E(X) = 9 \times 0.09 = 0.81$.

3.1.5 (a) $P(B(8, 0.5) = 5) = 0.2187$.

(b) $P(B(8, 1/6) = 1) = 0.3721$.

(c) $P(B(8, 1/6) = 0) = 0.2326$.

(d) $P(B(8, 2/3) \geq 6) = 0.4682$.

3.1.6 $P(B(10, 0.2) \geq 7) = 0.0009$.
$P(B(10, 0.5) \geq 7) = 0.1719$.

3.1.7 Let the random variable X be the number of employees taking sick leave. Then $X \sim B(180, 0.35)$. The *proportion* of the workforce who need to take sick leave is

$$Y = \frac{X}{180}.$$

Then

$$E(Y) = \frac{E(X)}{180} = \frac{180 \times 0.35}{180} = 0.35$$

and

$$Var(Y) = \frac{Var(X)}{180^2} = \frac{180 \times 0.35 \times 0.65}{180^2} = 0.0013.$$

In general the variance is

$$Var(Y) = \frac{Var(X)}{180^2} = \frac{180 \times p \times (1-p)}{180^2} = \frac{p \times (1-p)}{180}$$

which is maximized when $p = 0.5$.

3.2 The Geometric and Negative Binomial Distributions

3.2.1 (a) $P(X = 4) = (1 - 0.7)^3 \times 0.7 = 0.0189$.

(b) $P(X = 1) = (1 - 0.7)^0 \times 0.7 = 0.7$.

(c) $P(X \leq 5) = 1 - (1 - 0.7)^5 = 0.9976$.

(d) $P(X \geq 8) = 1 - P(X \leq 7) = (1 - 0.7)^7 = 0.0002$.

3.2.2 (a) $P(X = 5) = \binom{4}{2} \times (1 - 0.6)^2 \times 0.6^3 = 0.2074$.

(b) $P(X = 8) = \binom{7}{2} \times (1 - 0.6)^5 \times 0.6^3 = 0.0464$.

(c) $P(X \leq 7) = P(X = 3) + P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7)$ $= 0.9037$.

(d) $P(X \geq 7) = 1 - P(X = 3) - P(X = 4) - P(X = 5) - P(X = 6) = 0.1792$.

3.2.4 Notice that a negative binomial distribution with parameters p and r can be thought of as the number of trials up to and including the r^{th} success in a sequence of independent Bernoulli trials with a constant success probability p, which can be considered to be the number of trials up to and including the first success, plus the number of trials after the first success and up to and including the second success, plus the number of trials after the second success and up to and including the third success, and so on. Each of these r components has a geometric distribution with parameter p.

3.2.5 (a) Consider a geometric distribution with parameter $p = 0.09$.
$(1 - 0.09)^3 \times 0.09 = 0.0678$.

(b) Consider a negative binomial distribution with parameters $p = 0.09$ and $r = 3$.
$\binom{9}{2} \times (1 - 0.09)^7 \times 0.09^3 = 0.0136$.

(c) $\frac{1}{0.09} = 11.11$.

(d) $\frac{3}{0.09} = 33.33$.

3.2.6 (a) $\frac{1}{0.37} = 2.703$.

(b) $\frac{3}{0.37} = 8.108$.

(c) $P(X \leq 10) = 0.7794$ where the random variable X has a negative binomial distribution with parameters $p = 0.37$ and $r = 3$.

(d) $P(X = 10) = \binom{9}{2} \times (1 - 0.37)^7 \times 0.37^3 = 0.0718$.

3.4.7 $P(B(500, 0.005) \leq 3)$ can be approximated as

$$P(Poisson(500 \times 0.005) \leq 3) \;=\; P(Poisson(2.5) \leq 3)$$

$$= \frac{e^{-2.5} \times 2.5^0}{0!} + \frac{e^{-2.5} \times 2.5^1}{1!} - \frac{e^{-2.5} \times 2.5^2}{2!} = 0.7576.$$

3.4.8 $X \sim P(9.2)$.

(a) $P(X = 6) + P(X = 7) + P(X = 8) + P(X = 9) + P(X = 10) =$
$\frac{e^{-9.2} \times 9.2^6}{6!} + \frac{e^{-9.2} \times 9.2^7}{7!} + \frac{e^{-9.2} \times 9.2^8}{8!} + \frac{e^{-9.2} \times 9.2^9}{9!} + \frac{e^{-9.2} \times 9.2^{10}}{10!} =$
$0.0851 + 0.1118 + 0.1286 + 0.1315 + 0.1210 = 0.5780.$

(b) $P(X = 0) + P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) =$
$\frac{e^{-9.2} \times 9.2^0}{0!} + \frac{e^{-9.2} \times 9.2^1}{1!} + \frac{e^{-9.2} \times 9.2^2}{2!} + \frac{e^{-9.2} \times 9.2^3}{3!} + \frac{e^{-9.2} \times 9.2^4}{4!} =$
$0.0001 + 0.0009 + 0.0043 + 0.0131 + 0.0302 = 0.0486.$

H.W. 5

3.5 The Multinomial Distribution

3.5.1 (a) $$\frac{11!}{4! \times 5! \times 2!} \times 0.23^4 \times 0.48^5 \times 0.29^2 = 0.0416.$$

(b) $P(B(7, 0.23) < 3) = 0.7967$.

3.5.2 (a) $$\frac{15!}{3! \times 3! \times 9!} \times \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{2}{3}\right)^9 = 0.0558.$$

(b) $$\frac{15!}{3! \times 3! \times 4! \times 5!} \times \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{6}\right)^3 \times \left(\frac{1}{6}\right)^4 \times \left(\frac{1}{2}\right)^5 = 0.0065.$$

(c) $$\frac{15!}{2! \times 13!} \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^{13} = 0.2726.$$

The expected number of sixes is $\frac{15}{6} = 2.5$.

3.5.3 (a) $$\frac{8!}{2! \times 5! \times 1!} \times 0.09^2 \times 0.79^5 \times 0.12^1 = 0.0502.$$

(b) $$\frac{8!}{1! \times 5! \times 2!} \times 0.09^1 \times 0.79^5 \times 0.12^2 = 0.0670.$$

(c) $P(B(8, 0.09) \geq 2) = 0.1577$.

The expected number of misses is $8 \times 0.12 = 0.96$.

3.5.4 The expected number of dead seedlings is $22 \times 0.08 = 1.76$, the expected number of slow growth seedlings is $22 \times 0.19 = 4.18$, the expected number of medium growth seedlings is $22 \times 0.42 = 9.24$, and the expected number of strong growth seedlings is $22 \times 0.31 = 6.82$.

(a) $$\frac{22!}{3! \times 4! \times 6! \times 9!} \times 0.08^3 \times 0.19^4 \times 0.42^6 \times 0.31^9 = 0.0029.$$

(b) $$\frac{22!}{5! \times 5! \times 5! \times 7!} \times 0.08^5 \times 0.19^5 \times 0.42^5 \times 0.31^7 = 0.00038.$$

(c) $P(B(22, 0.08) \leq 2) = 0.7442$.

3.6 Supplementary Problems

3.6.1 (a) $P(B(18, 0.085) \geq 3) = 1 - P(B(18, 0.085) \leq 2) = 0.1931$.

(b) $P(B(18, 0.085) \leq 1) = 0.5401$.

(c) $18 \times 0.085 = 1.53$.

3.6.2 $P(B(13, 0.4) \geq 3) = 1 - P(B(13, 0.4) \leq 2) = 0.9421$.

The expected number of cells is $13 + (13 \times 0.4) = 18.2$.

3.6.3 (a)

$$\frac{8!}{2! \times 3! \times 3!} \times 0.40^2 \times 0.25^3 \times 0.35^3 = 0.0600.$$

(b)

$$\frac{8!}{3! \times 1! \times 4!} \times 0.40^3 \times 0.25^1 \times 0.35^4 = 0.0672.$$

(c) $P(B(8, 0.35) \leq 2) = 0.4278$.

3.6.4 (a)

$$P(X = 0) = \frac{e^{-2/3} \times (2/3)^0}{0!} = 0.5134.$$

(b)

$$P(X = 1) = \frac{e^{-2/3} \times (2/3)^1}{1!} = 0.3423.$$

(c) $P(X \geq 3) = 1 - P(X \leq 2) = 0.0302$.

3.6.5

$$P(X = 2) = \frac{e^{-3.3} \times (3.3)^2}{2!} = 0.2008.$$

$P(X \geq 6) = 1 - P(X \leq 5) = 0.1171$.

3.6.6 (a) Consider a negative binomial distribution with parameters $p = 0.55$ and $r = 4$.

(b) $P(X = 7) = \binom{6}{3} \times (1 - 0.55)^3 \times 0.55^4 = 0.1668$.

(c) $P(X = 6) = \binom{5}{3} \times (1 - 0.55)^2 \times 0.55^4 = 0.1853$.

(d) The probability that team A wins the series in game 5 is

$\binom{4}{3} \times (1 - 0.55)^1 \times 0.55^4 = 0.1647$.

The probability that team B wins the series in game 5 is

$\binom{4}{3} \times (1 - 0.45)^1 \times 0.45^4 = 0.0902$.

The probability that the series is over after game five is $0.1647 + 0.0902 = 0.2549$.

(e) The probability that team A wins the series in game 4 is $0.55^4 = 0.0915$.
The probability that team A wins the series is $0.0915 + 0.1647 + 0.1853 + 0.1668 = 0.6083$.

3.6.7 (a) Consider a negative binomial distribution with parameters $p = 0.58$ and $r = 3$.
$P(X = 9) = \binom{8}{2} \times (1 - 0.58)^6 \times 0.58^3 = 0.0300$.

(b) Consider a negative binomial distribution with parameters $p = 0.42$ and $r = 4$.
$P(X \leq 7) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) = 0.3294$.

3.6.8
$$P(\mathit{two\ red\ balls}|\mathit{head}) = \frac{\binom{6}{2} \times \binom{5}{1}}{\binom{11}{3}} = \frac{5}{11}.$$

$$P(\mathit{two\ red\ balls}|\mathit{tail}) = \frac{\binom{5}{2} \times \binom{6}{1}}{\binom{11}{3}} = \frac{4}{11}.$$

Then

$$P(\mathit{two\ red\ balls}) = (P(\mathit{head}) \times P(\mathit{two\ red\ balls}|\mathit{head}))$$
$$+ (P(\mathit{tail}) \times P(\mathit{two\ red\ balls}|\mathit{tail}))$$
$$= \left(0.5 \times \frac{5}{11}\right) + \left(0.5 \times \frac{4}{11}\right) = \frac{9}{22}$$

and

$$P(\mathit{head}|\mathit{two\ red\ balls}) = \frac{P(\mathit{head\ and\ two\ red\ balls})}{P(\mathit{two\ red\ balls})}$$
$$= \frac{P(\mathit{head}) \times P(\mathit{two\ red\ balls}|\mathit{head})}{P(\mathit{two\ red\ balls})} = \frac{5}{9}.$$

3.6.9 Using the hypergeometric distribution, the answer is

$$P(X = 0) + P(X = 1) = \frac{\binom{36}{5} \times \binom{4}{0}}{\binom{40}{5}} + \frac{\binom{36}{4} \times \binom{4}{1}}{\binom{40}{5}} = 0.9310.$$

for $\theta \leq x \leq \infty$.

(a) $P(X \leq 0) = F(0) = \frac{1}{2}e^{-3(2-0)} = 0.0012$.

(b) $P(X \geq 1) = 1 - F(1) = 1 - \frac{1}{2}e^{-3(2-1)} = 0.9751$.

4.2.6 (a) $E(X) = \frac{1}{2} = 0.5$.

(b) $P(X \geq 1) = 1 - F(1) = 1 - (1 - e^{-2\times 1}) = e^{-2} = 0.1353$.

(c) A Poisson distribution with parameter $2 \times 3 = 6$.

(d)
$$\begin{aligned} P(X \leq 4) &= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4) \\ &= \frac{e^{-6}\times 6^0}{0!} + \frac{e^{-6}\times 6^1}{1!} + \frac{e^{-6}\times 6^2}{2!} + \frac{e^{-6}\times 6^3}{3!} + \frac{e^{-6}\times 6^4}{4!} = 0.2851. \end{aligned}$$

4.2.7 (a) $\lambda = 1.8$.

(b) $E(X) = \frac{1}{1.8} = 0.5556$.

(c) $P(X \geq 1) = 1 - F(1) = 1 - (1 - e^{-1.8\times 1}) = e^{-1.8} = 0.1653$.

(d) A Poisson distribution with parameter $1.8 \times 4 = 7.2$.

(e)
$$\begin{aligned} P(X \geq 4) &= 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3) \\ &= 1 - \frac{e^{-7.2}\times 7.2^0}{0!} - \frac{e^{-7.2}\times 7.2^1}{1!} - \frac{e^{-7.2}\times 7.2^2}{2!} - \frac{e^{-7.2}\times 7.2^3}{3!} = 0.9281. \end{aligned}$$

4.2.8 (a) Solving $F(5) = 1 - e^{-\lambda\times 5} = 0.90$ gives $\lambda = 0.4605$.

(b) $F(3) = 1 - e^{-0.4605\times 3} = 0.75$.

H.W. #6

5.1.6 Solving $P(X \leq 10) = \Phi(\frac{10-\mu}{\sigma}) = 0.55$ and $P(X \leq 0) = \Phi(\frac{0-\mu}{\sigma}) = 0.4$ gives $\mu = 6.6845$ and $\sigma = 26.3845$.

5.1.7 $P(X \leq \mu + \sigma z_\alpha) = \Phi(\frac{\mu+\sigma z_\alpha - \mu}{\sigma}) = \Phi(z_\alpha) = 1 - \alpha$.

$P(\mu - \sigma z_{\alpha/2} \leq X \leq \mu + \sigma z_{\alpha/2}) = \Phi(\frac{\mu+\sigma z_{\alpha/2}-\mu}{\sigma}) - \Phi(\frac{\mu-\sigma z_{\alpha/2}-\mu}{\sigma}) = \Phi(z_{\alpha/2}) - \Phi(-z_{\alpha/2})$
$= 1 - \alpha/2 - \alpha/2 = 1 - \alpha$.

5.1.8 Solving $\Phi(x) = 0.75$ gives $x = 0.6745$.

Solving $\Phi(x) = 0.25$ gives $x = -0.6745$.

The interquartile range of a $N(0,1)$ distribution is $0.6745 - (-0.6745) = 1.3490$.

The interquartile range of a $N(\mu, \sigma^2)$ distribution is $1.3490 \times \sigma$.

5.1.9 (a) $P(N(3.00, 0.12^2) \geq 3.2) = 0.0478$.

(b) $P(N(3.00, 0.12^2) \leq 2.7) = 0.0062$.

(c) Solving $P(3.00 - c \leq N(3.00, 0.12^2) \leq 3.00 + c) = 0.99$ gives $c = 0.12 \times z_{0.005}$
$= 0.12 \times 2.5758 = 0.3091$.

5.1.10 (a) $P(N(1.03, 0.014^2) \leq 1) = 0.0161$.

(b) $P(N(1.05, 0.016^2) \leq 1) = 0.0009$.
There is a decrease in the proportion of underweight packets.

(c) The expected excess weight is $\mu - 1$ which is 0.03 and 0.05.

5.1.11 (a) Solving $P(N(4.3, 0.12^2) \leq x) = 0.75$ gives $x = 4.3809$.
Solving $P(N(4.3, 0.12^2) \leq x) = 0.25$ gives $x = 4.2191$.

(b) Solving $P(4.3 - c \leq N(4.3, 0.12^2) \leq 4.3 + c) = 0.80$ gives
$c = 0.12 \times z_{0.10} = 0.12 \times 1.2816 = 0.1538$.

5.1.12 (a) $P(N(0.0046, 9.6 \times 10^{-8}) \leq 0.005) = 0.9017$.

(b) $P(0.004 \leq N(0.0046, 9.6 \times 10^{-8}) \leq 0.005) = 0.8753$.

5.2 Linear Combinations of Normal Random Variables

5.2.1 (a) $P(N(3.2 + (-2.1), 6.5 + 3.5) \geq 0) = 0.6360$.

(b) $P(N(3.2 + (-2.1) - (2 \times 12.0), 6.5 + 3.5 + (2^2 \times 7.5)) \leq -20) = 0.6767$.

(c) $P(N((3 \times 3.2) + (5 \times (-2.1)), (3^2 \times 6.5) + (5^2 \times 3.5)) \geq 1) = 0.4375$.

(d) $P(N((4 \times 3.2) - (4 \times (-2.1)) + (2 \times 12.0), (4^2 \times 6.5) + (4^2 \times 3.5) + (2^2 \times 7.5)) \leq 25) = 0.0714$.

(e) $P(|N(3.2 + (6 \times (-2.1)) + 12.0, 6.5 + (6^2 \times 3.5) + 7.5)| \geq 2) = 0.8689$.

(f) $P(|N((2 \times 3.2) - (-2.1) - 6, (2^2 \times 6.5) + 3.5)| \leq 1) = 0.1315$.

5.2.2 (a) $P(N(-1.9 - 3.3, 2.2 + 1.7) \geq -3) = 0.1326$.

(b) $P(N((2 \times (-1.9)) + (3 \times 3.3) + (4 \times 0.8), (2^2 \times 2.2) + (3^2 \times 1.7) + (4^2 \times 0.2)) \leq 10) = 0.5533$.

(c) $P(N((3 \times 3.3) - 0.8, (3^2 \times 1.7) + 0.2) \leq 8) = 0.3900$.

(d) $P(N((2 \times (-1.9)) - (2 \times 3.3) + (3 \times 0.8), (2^2 \times 2.2) + (2^2 \times 1.7) + (3^2 \times 0.2)) \leq -6) = 0.6842$.

(e) $P(|N(-1.9 + 3.3 - 0.8, 2.2 + 1.7 + 0.2)| \geq 1.5) = 0.4781$.

(f) $P(|N((4 \times (-1.9)) - 3.3 + 10, (4^2 \times 2.2) + 1.7)| \leq 0.5) = 0.0648$.

5.2.3 (a) $\Phi(0.5) - \Phi(-0.5) = 0.3830$.

(b) $P(|N(0, \frac{1}{8})| \leq 0.5) = 0.8428$.

(c) Need $0.5\sqrt{n} \geq z_{0.005} = 2.5758$ which is satisfied for $n \geq 27$.

5.2.4 (a) $N(4.3 + 4.3, 0.12^2 + 0.12^2) = N(8.6, 0.0288)$.

(b) $N(4.3, \frac{0.12^2}{12}) = N(4.3, 0.0012)$.

5.2.10 (a) Let the random variables X_i be the widths of the components. Then $P(X_1 + X_2 + X_3 + X_4 \leq 10402.5) = P(N(4 \times 2600, 4 \times 0.6^2) \leq 10402.5) = \Phi\left(\frac{10402.5-10400}{1.2}\right) = \Phi(2.083) = 0.9814$.

(b) Let the random variable Y be the width of the slot. Then $P(X_1 + X_2 + X_3 + X_4 - Y \leq 0) = P(N(4 \times 2600 - 10402.5, 4 \times 0.6^2 + 0.4^2) \leq 0)$ $= \Phi\left(\frac{2.5}{1.2649}\right) = \Phi(1.976) = 0.9759$.

H. W. #7 (Bonus)

5.2 Linear Combinations of Normal Random Variables

5.2.1 (a) $P(N(3.2 + (-2.1), 6.5 + 3.5) \geq 0) = 0.6360$.

(b) $P(N(3.2 + (-2.1) - (2 \times 12.0), 6.5 + 3.5 + (2^2 \times 7.5)) \leq -20) = 0.6767$.

(c) $P(N((3 \times 3.2) + (5 \times (-2.1)), (3^2 \times 6.5) + (5^2 \times 3.5)) \geq 1) = 0.4375$.

(d) $P(N((4 \times 3.2) - (4 \times (-2.1)) + (2 \times 12.0), (4^2 \times 6.5) + (4^2 \times 3.5) + (2^2 \times 7.5)) \leq 25) = 0.0714$.

(e) $P(|N(3.2 + (6 \times (-2.1)) + 12.0, 6.5 + (6^2 \times 3.5) + 7.5)| \geq 2) = 0.8689$.

(f) $P(|N((2 \times 3.2) - (-2.1) - 6, (2^2 \times 6.5) + 3.5)| \leq 1) = 0.1315$.

5.2.2 (a) $P(N(-1.9 - 3.3, 2.2 + 1.7) \geq -3) = 0.1326$.

(b) $P(N((2 \times (-1.9)) + (3 \times 3.3) + (4 \times 0.8), (2^2 \times 2.2) + (3^2 \times 1.7) + (4^2 \times 0.2)) \leq 10) = 0.5533$.

(c) $P(N((3 \times 3.3) - 0.8, (3^2 \times 1.7) + 0.2) \leq 8) = 0.3900$.

(d) $P(N((2 \times (-1.9)) - (2 \times 3.3) + (3 \times 0.8), (2^2 \times 2.2) + (2^2 \times 1.7) + (3^2 \times 0.2)) \leq -6) = 0.6842$.

(e) $P(|N(-1.9 + 3.3 - 0.8, 2.2 + 1.7 + 0.2)| \geq 1.5) = 0.4781$.

(f) $P(|N((4 \times (-1.9)) - 3.3 + 10, (4^2 \times 2.2) + 1.7)| \leq 0.5) = 0.0648$.

5.2.3 (a) $\Phi(0.5) - \Phi(-0.5) = 0.3830$.

(b) $P(|N(0, \frac{1}{8})| \leq 0.5) = 0.8428$.

(c) Need $0.5\sqrt{n} \geq z_{0.005} = 2.5758$ which is satisfied for $n \geq 27$.

— 5.2.4 (a) $N(4.3 + 4.3, 0.12^2 + 0.12^2) = N(8.6, 0.0288)$.

(b) $N(4.3, \frac{0.12^2}{12}) = N(4.3, 0.0012)$.

(c) Need $z_{0.0015} \times \frac{0.12}{\sqrt{n}} = 2.9677 \times \frac{0.12}{\sqrt{n}} \leq 0.05$ which is satisfied for $n \geq 51$.

5.2.5 $P(144 \leq N(37+37+24+24+24, 0.49+0.49+0.09+0.09+0.09) \leq 147) = 0.7777$.

5.2.6 (a) $Var(Y) = (p^2 \times \sigma_1^2) + ((1-p)^2 \times \sigma_2^2)$.
The minimum variance is

$$\frac{1}{\frac{1}{\sigma_1^2} + \frac{1}{\sigma_2^2}} = \frac{\sigma_1^2\,\sigma_2^2}{\sigma_1^2 + \sigma_2^2}.$$

(b) In this case

$$Var(Y) = \sum_{i=1}^{n} p_i^2\,\sigma_i^2.$$

The variance is minimized with

$$p_i = \frac{\frac{1}{\sigma_i^2}}{\frac{1}{\sigma_1^2} + \ldots + \frac{1}{\sigma_n^2}}$$

and the minimum variance is

$$\frac{1}{\frac{1}{\sigma_1^2} + \ldots + \frac{1}{\sigma_n^2}}.$$

5.2.7 (a) $1.05y + 1.05(1000 - y) = \$1050$.

(b) $0.0002y^2 + 0.0003(1000 - y)^2$.

(c) The variance is minimized with $y = 600$ and the minimum variance is 120.
$P(N(1050, 120) \geq 1060) = 0.1807$.

5.2.8 (a) $P(N(3.00 + 3.00 + 3.00, 0.12^2 + 0.12^2 + 0.12^2) \geq 9.50) = 0.0081$.

(b) $P(N(3.00, \frac{0.12^2}{7}) \leq 3.10) = 0.9863$.

5.2.9 (a) $N(22 \times 1.03, 22 \times 0.014^2) = N(22.66, 4.312 \times 10^{-3})$.

(b) Solving $P(N(22.66, 4.312 \times 10^{-3}) \leq x) = 0.75$ gives $x = 22.704$.
Solving $P(N(22.66, 4.312 \times 10^{-3}) \leq x) = 0.25$ gives $x = 22.616$.

5.3 Approximating Distributions with the Normal Distribution

5.3.1 (a) The exact probability is 0.3823.
The normal approximation is $1 - \Phi(\frac{8-0.5-(10\times0.7)}{\sqrt{10\times0.7\times0.3}}) = 0.3650$.

(b) The exact probability is 0.9147.
The normal approximation is $\Phi(\frac{7+0.5-(15\times0.3)}{\sqrt{15\times0.3\times0.7}}) - \Phi(\frac{1+0.5-(15\times0.3)}{\sqrt{15\times0.3\times0.7}}) = 0.9090$.

(c) The exact probability is 0.7334.
The normal approximation is $\Phi(\frac{4+0.5-(9\times0.4)}{\sqrt{9\times0.4\times0.6}}) = 0.7299$.

(d) The exact probability is 0.6527.
The normal approximation is $\Phi(\frac{11+0.5-(14\times0.6)}{\sqrt{14\times0.6\times0.4}}) - \Phi(\frac{7+0.5-(14\times0.6)}{\sqrt{14\times0.6\times0.4}}) = 0.6429$.

5.3.2 (a) The exact probability is 0.0106.
The normal approximation is $1 - \Phi(\frac{7-0.5-(10\times0.3)}{\sqrt{10\times0.3\times0.7}}) = 0.0079$.

(b) The exact probability is 0.6160.
The normal approximation is $\Phi(\frac{12+0.5-(21\times0.5)}{\sqrt{21\times0.5\times0.5}}) - \Phi(\frac{8+0.5-(21\times0.5)}{\sqrt{21\times0.5\times0.5}}) = 0.6172$.

(c) The exact probability is 0.9667.
The normal approximation is $\Phi(\frac{3+0.5-(7\times0.2)}{\sqrt{7\times0.2\times0.8}}) = 0.9764$.

(d) The exact probability is 0.3410.
The normal approximation is $\Phi(\frac{11+0.5-(12\times0.65)}{\sqrt{12\times0.65\times0.35}}) - \Phi(\frac{8+0.5-(12\times0.65)}{\sqrt{12\times0.65\times0.35}}) = 0.3233$.

5.3.3 The required probability is

$$\Phi\left(0.02\sqrt{n} + \frac{1}{\sqrt{n}}\right) - \Phi\left(-0.02\sqrt{n} - \frac{1}{\sqrt{n}}\right)$$

which is equal to
0.2358 for $n = 100$,
0.2764 for $n = 200$,
0.3772 for $n = 500$,
0.4934 for $n = 1{,}000$, and
0.6408 for $n = 2{,}000$.

5.3.4 (a) $\Phi(\frac{180+0.5-(1{,}000\times1/6)}{\sqrt{1{,}000\times1/6\times5/6}}) - \Phi(\frac{149+0.5-(1{,}000\times1/6)}{\sqrt{1{,}000\times1/6\times5/6}}) = 0.8072$.

(b) It is required that

$$1 - \Phi\left(\frac{50 - 0.5 - n/6}{\sqrt{n \times 1/6 \times 5/6}}\right) \geq 0.99$$

which is satisfied for $n \geq 402$.

5.3.5 (a) A normal distribution can be used with $\mu = 500 \times 2.4 = 1,200$ and $\sigma^2 = 500 \times 2.4 = 1,200$.

(b) $P(N(1200, 1200) \geq 1250) = 0.0745$.

5.3.6 The normal approximation is $1 - \Phi(\frac{135-0.5-(15,000\times 1/125)}{\sqrt{15,000\times 1/125\times 124/125}}) = 0.0919$.

5.3.7 The normal approximation is $\Phi(\frac{200+0.5-(250,000\times 0.0007)}{\sqrt{250,000\times 0.0007\times 0.9993}}) = 0.9731$.

5.3.8 (a) The normal approximation is $1 - \Phi(\frac{30-0.5-(60\times 0.25)}{\sqrt{60\times 0.25\times 0.75}}) \simeq 0$.

(b) It is required that $P(B(n, 0.25) \leq 0.35n) \geq 0.99$ which using the normal approximation can be written

$$\Phi\left(\frac{0.35n + 0.5 - 0.25n}{\sqrt{n \times 0.25 \times 0.75}}\right) \geq 0.99.$$

This is satisfied for $n \geq 92$.

5.3.9 The yearly take can be approximated by a normal distribution with $\mu = 365 \times \frac{5}{0.9} = 2{,}027.8$ and $\sigma^2 = 365 \times \frac{5}{0.9^2} = 2{,}253.1$.
$P(N(2027.8, 2253.1) \geq 2,000) = 0.7210$.

5.3.10 The normal approximation is $P(N(1500 \times 0.6, 1500 \times 0.6 \times 0.4) \geq 925 - 0.5) = 1 - \Phi(1.291) = 0.0983$.

5.3.11 The expectation of the strength of a chemical solution is $E(X) = \frac{18}{18+11} = 0.6207$ and the variance is
$Var(X) = \frac{18\times 11}{(18+11)^2(18+11+1)} = 0.007848$.

Using the central limit theorem the required probability can be estimated as
$P(0.60 \leq N(0.6207, 0.007848/20) \leq 0.65) = \Phi(1.479) - \Phi(-1.045) = 0.7824$.

H. W. #B

6.1.7 The population may be all bricks shipped by that company, or just the bricks in that delivery. The random selection of the sample should ensure that it is representative of that delivery of bricks. That delivery of bricks may not be representative of all the deliveries from that company.

6.1.8 The population is all car panels spray painted by the machine. The selection method of the sample should ensure that it is representative.

6.1.9 The population is all plastic panels made by the machine. If the 80 sample panels are selected in some random manner then they should be representative of the population.

6.3 Sample Statistics

Note: The sample statistics for the problems in this section depend upon whether any observations have been removed as outliers. To avoid confusion, the answers given here assume that **no** observations have been removed.

The trimmed means given here are those obtained by removing the largest 5% and the smallest 5% of the data observations.

6.3.1 The sample mean is $\bar{x} = 155.95$.

The sample median is 159.

The sample trimmed mean is 156.50.

The sample standard deviation is $s = 18.43$.

The upper sample quartile is 169.5.

The lower sample quartile is 143.25.

6.3.2 The sample mean is $\bar{x} = 1.2006$.

The sample median is 1.2010.

The sample trimmed mean is 1.2007.

The sample standard deviation is $s = 0.0291$.

The upper sample quartile is 1.2097.

The lower sample quartile is 1.1890.

6.3.3 The sample mean is $\bar{x} = 37.08$.

The sample median is 35.

The sample trimmed mean is 36.35.

The sample standard deviation is $s = 8.32$.

The upper sample quartile is 40.

The lower sample quartile is 33.5.

6.3.4 The sample mean is $\bar{x} = 3.567$.

The sample median is 3.5.

The sample trimmed mean is 3.574.

The sample standard deviation is $s = 1.767$.

The upper sample quartile is 5.

The lower sample quartile is 2.

6.5 Supplementary Problems

6.5.1 The population from which the sample is drawn should be all the birds on the island. However, the sample may not be representative if some species are more likely to be observed than others.

It appears that the grey markings are the most common followed by the black markings.

6.5.2 There do not appear to be any seasonal effects although there may possibly be a correlation from one month to the next.

The sample mean is $\bar{x} = 17.79$.

The sample median is 17.

The sample trimmed mean is 17.36.

The sample standard deviation is $s = 6.16$.

The upper sample quartile is 21.75.

The lower sample quartile is 14.

6.5.3 One question of interest in interpreting the data set is whether the month of sampling is representative of other months.

The sample is skewed.

The most frequent data value (the sample mode) is one error.

The sample mean is $\bar{x} = 1.633$.

The sample median is 1.5.

The sample trimmed mean is 1.615.

The sample standard deviation is $s = 0.999$.

The upper sample quartile is 2.

The lower sample quartile is 1.

6.5.4 The population could be all adult males who visit the clinic. This could be representative of all adult males in the population unless there is something special about the clientele of the clinic.

The largest observation 75.9 looks like an outlier on a histogram but may be a valid observation.

The sample mean is $\bar{x} = 69.618$.

The sample median is 69.5.

The sample trimmed mean is 69.513.

The sample standard deviation is $s = 1.523$.

The upper sample quartile is 70.275.

Chapter 7

Statistical Estimation and Sampling Distributions

7.2 Properties of Point Estimates

7.2.1 (a) bias$(\hat{\mu}_1) = 0$. The point estimate $\hat{\mu}_1$ is unbiased.
bias$(\hat{\mu}_2) = 0$. The point estimate $\hat{\mu}_2$ is unbiased.
bias$(\hat{\mu}_3) = 9 - \frac{\mu}{2}$.

(b) $Var(\hat{\mu}_1) = 6.25$.
$Var(\hat{\mu}_2) = 9.0625$.
$Var(\hat{\mu}_3) = 1.9444$. The point estimate $\hat{\mu}_3$ has the smallest variance.

(c) $MSE(\hat{\mu}_1) = 6.25$.
$MSE(\hat{\mu}_2) = 9.0625$.
$MSE(\hat{\mu}_3) = 1.9444 + (9 - \frac{\mu}{2})^2$. This is equal to 26.9444 when $\mu = 8$.

7.2.2 (a) bias$(\hat{\mu}_1) = 0$. The point estimate $\hat{\mu}_1$ is unbiased.
bias$(\hat{\mu}_2) = -0.217\mu$.
bias$(\hat{\mu}_3) = 2 - \frac{\mu}{4}$.

(b) $Var(\hat{\mu}_1) = 4.444$.
$Var(\hat{\mu}_2) = 2.682$. The point estimate $\hat{\mu}_2$ has the smallest variance.
$Var(\hat{\mu}_3) = 2.889$.

(c) $MSE(\hat{\mu}_1) = 4.444$.
$MSE(\hat{\mu}_2) = 2.682 + 0.0469\mu^2$. This is equal to 3.104 when $\mu = 3$.
$MSE(\hat{\mu}_3) = 2.889 + (2 - \frac{\mu}{4})^2$. This is equal to 4.452 when $\mu = 3$.

7.2.3 (a) $Var(\hat{\mu}_1) = 2.5$.

(b) The value $p = 0.6$ produces the smallest variance which is $Var(\hat{\mu}) = 2.4$.

(c) The relative efficiency is $\frac{2.4}{2.5} = 0.96$.

7.2.4 (a) $Var(\hat{\mu}_1) = 2$.

(b) The value $p = 0.875$ produces the smallest variance which is $Var(\hat{\mu}) = 0.875$.

(c) The relative efficiency is $\frac{0.875}{2} = 0.4375$.

7.2.5 (a) $a_1 + \ldots + a_n = 1$.

(b) $a_1 = \ldots = a_n = \frac{1}{n}$.

7.2.6 $MSE(\hat{\theta}_1) = 0.02\,\theta^2 + (0.13\,\theta)^2 = 0.0369\,\theta^2$.
$MSE(\hat{\theta}_2) = 0.07\,\theta^2 + (0.05\,\theta)^2 = 0.0725\,\theta^2$.
$MSE(\hat{\theta}_3) = 0.005\,\theta^2 + (0.24\,\theta)^2 = 0.0626\,\theta^2$.
The point estimate $\hat{\theta}_1$ has the smallest mean square error.

7.2.7 $\text{bias}(\hat{\mu}) = \frac{\mu_0 - \mu}{2}$.
$Var(\hat{\mu}) = \frac{\sigma^2}{4}$.
$MSE(\hat{\mu}) = \frac{\sigma^2}{4} + \frac{(\mu_0 - \mu)^2}{4}$.
$MSE(X) = \sigma^2$.

7.2.8 (a) $\text{bias}(\hat{p}) = -\frac{p}{11}$.

(b) $Var(\hat{p}) = \frac{10\,p\,(1-p)}{121}$.

(c) $MSE(\hat{p}) = \frac{10\,p\,(1-p)}{121} + (\frac{p}{11})^2 = \frac{10p - 9p^2}{121}$.

(d) $\text{bias}(X/10) = 0$.
$Var(X/10) = \frac{p(1-p)}{10}$.
$MSE(X/10) = \frac{p(1-p)}{10}$.

H. W. #9

ENGR 803

Chapter 8

Inferences on a Population Mean

8.1 Confidence Intervals

8.1.1 With $t_{0.025,30} = 2.042$ the confidence interval is
$(53.42 - \frac{2.042\times3.05}{\sqrt{31}}, 53.42 + \frac{2.042\times3.05}{\sqrt{31}}) = (52.30, 54.54)$.

8.1.2 With $t_{0.005,40} = 2.704$ the confidence interval is
$(3.04 - \frac{2.704\times0.124}{\sqrt{41}}, 3.04 + \frac{2.704\times0.124}{\sqrt{41}}) = (2.99, 3.09)$.

The confidence interval does not contain the value 2.90 and so it is not a plausible value for the mean glass thickness.

8.1.3 At 90% confidence $t_{0.05,19} = 1.729$ and the confidence interval is
$(436.5 - \frac{1.729\times11.90}{\sqrt{20}}, 436.5 + \frac{1.729\times11.90}{\sqrt{20}}) = (431.9, 441.1)$.

At 95% confidence $t_{0.025,19} = 2.093$ and the confidence interval is
$(436.5 - \frac{2.093\times11.90}{\sqrt{20}}, 436.5 + \frac{2.093\times11.90}{\sqrt{20}}) = (430.9, 442.1)$.

At 99% confidence $t_{0.005,19} = 2.861$ and the confidence interval is
$(436.5 - \frac{2.861\times11.90}{\sqrt{20}}, 436.5 + \frac{2.861\times11.90}{\sqrt{20}}) = (428.9, 444.1)$.

Even the 99% confidence level confidence interval does not contain the value 450.0 and so it is not a plausible value for the average breaking strength.

8.1.4 With $t_{0.005,15} = 2.947$ the confidence interval is
$(1.053 - \frac{2.947\times0.058}{\sqrt{16}}, 1.053 + \frac{2.947\times0.058}{\sqrt{16}}) = (1.010, 1.096)$.

The confidence interval contains the value 1.025 and so it is a plausible value for the average weight.

8.1.5 With $z_{0.025} = 1.960$ the confidence interval is
$(0.0328 - \frac{1.960 \times 0.015}{\sqrt{28}}, 0.0328 + \frac{1.960 \times 0.015}{\sqrt{28}}) = (0.0272, 0.0384)$.

8.1.6 At 90% confidence $z_{0.05} = 1.645$ and the confidence interval is
$(19.50 - \frac{1.645 \times 1.0}{\sqrt{10}}, 19.50 + \frac{1.645 \times 1.0}{\sqrt{10}}) = (18.98, 20.02)$.

At 95% confidence $z_{0.025} = 1.960$ and the confidence interval is
$(19.50 - \frac{1.960 \times 1.0}{\sqrt{10}}, 19.50 + \frac{1.960 \times 1.0}{\sqrt{10}}) = (18.88, 20.12)$.

At 99% confidence $z_{0.005} = 2.576$ and the confidence interval is
$(19.50 - \frac{2.576 \times 1.0}{\sqrt{10}}, 19.50 + \frac{2.576 \times 1.0}{\sqrt{10}}) = (18.69, 20.31)$.

Even the 90% confidence level confidence interval contains the value 20.0 and so it is a plausible value for the average resilient modulus.

8.1.7 With $t_{0.025,n-1} \simeq 2.0$ a sufficient sample size can be estimated as

$$n \geq 4 \times \left(\frac{t_{0.025,n-1}\, s}{L_0}\right)^2 = 4 \times \left(\frac{2.0 \times 10.0}{5}\right)^2 = 64.$$

A sample size of about $n = 64$ should be sufficient.

8.1.8 With $t_{0.005,n-1} \simeq 3.0$ a sufficient sample size can be estimated as

$$n \geq 4 \times \left(\frac{t_{0.005,n-1}\, s}{L_0}\right)^2 = 4 \times \left(\frac{3.0 \times 0.15}{0.2}\right)^2 = 20.25.$$

A sample size slightly larger than 20 should be sufficient.

8.1.9 A total sample size of

$$n \geq 4 \times \left(\frac{t_{0.025,n_1-1}\, s}{L_0}\right)^2 = 4 \times \left(\frac{2.042 \times 3.05}{2.0}\right)^2 = 38.8$$

is required. Therefore an additional sample of at least $39 - 31 = 8$ observations should be sufficient.

8.1.10 A total sample size of

$$n \geq 4 \times \left(\frac{t_{0.005,n_1-1}\, s}{L_0}\right)^2 = 4 \times \left(\frac{2.704 \times 0.124}{0.05}\right)^2 = 179.9$$

is required. Therefore an additional sample of at least $180 - 41 = 139$ observations should be sufficient.

8.2 Hypothesis Testing

8.2.1 (a) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{18} \times (57.74 - 55.0)}{11.2} = 1.04.$$

The p-value is

$$2 \times P(X \geq 1.04) = 0.313$$

where the random variable X has a t distribution with $18 - 1 = 17$ degrees of freedom.

(b) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{18} \times (57.74 - 65.0)}{11.2} = -2.75.$$

The p-value is

$$P(X \leq -2.75) = 0.0068$$

where the random variable X has a t distribution with $18 - 1 = 17$ degrees of freedom.

8.2.2 (a) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{39} \times (5{,}532 - 5{,}680)}{287.8} = -3.21.$$

The p-value is

$$2 \times P(X \geq 3.21) = 0.003$$

where the random variable X has a t distribution with $39 - 1 = 38$ degrees of freedom.

(b) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{39} \times (5{,}532 - 5{,}450)}{287.8} = 1.78.$$

The p-value is

$$P(X \geq 1.78) = 0.042$$

where the random variable X has a t distribution with $39 - 1 = 38$ degrees of freedom.

8.2.3 (a) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x} - \mu_0)}{\sigma} = \frac{\sqrt{13} \times (2.879 - 3.0)}{0.325} = -1.34.$$

The p-value is

$$2 \times \Phi(-1.34) = 0.180.$$

(b) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma} = \frac{\sqrt{13} \times (2.879 - 3.1)}{0.325} = -2.45.$$

The p-value is

$$\Phi(-2.45) = 0.007.$$

8.2.4 (a) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma} = \frac{\sqrt{44} \times (87.90 - 90.0)}{5.90} = -2.36.$$

The p-value is

$$2 \times \Phi(-2.36) = 0.018.$$

(b) The test statistic is

$$z = \frac{\sqrt{n}(\bar{x}-\mu_0)}{\sigma} = \frac{\sqrt{44} \times (87.90 - 86.0)}{5.90} = 2.14.$$

The p-value is

$$1 - \Phi(2.14) = 0.016.$$

8.2.5 (a) The critical point is $t_{0.05,40} = 1.684$ and the null hypothesis is accepted when $|t| \leq 1.684$.

(b) The critical point is $t_{0.005,40} = 2.704$ and the null hypothesis is rejected when $|t| > 2.704$.

(c) The test statistic is

$$t = \frac{\sqrt{n}(\bar{x}-\mu_0)}{s} = \frac{\sqrt{41} \times (3.04 - 3.00)}{0.124} = 2.066.$$

The null hypothesis is rejected at size $\alpha = 0.10$ and accepted at size $\alpha = 0.01$.

(d) The p-value is

$$2 \times P(X \geq 2.066) = 0.045$$

where the random variable X has a t distribution with $41 - 1 = 40$ degrees of freedom.

8.2.17 Consider the hypothesis testing problem

$$H_0 : \mu \geq 13 \quad \text{versus} \quad H_A : \mu < 13.$$

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{90} \times (12.211 - 13.000)}{2.629} = -2.85.$$

The p-value is

$$P(X \leq -2.85) = 0.0027$$

where the random variable X has a t distribution with $90-1 = 89$ degrees of freedom.

There is sufficient evidence to conclude that the average number of calls taken per minute is less than 13 so that the manager's claim is false.

8.2.18 Consider the hypothesis testing problem

$$H_0 : \mu = 1.1 \quad \text{versus} \quad H_A : \mu \neq 1.1.$$

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{125} \times (1.11059 - 1.10000)}{0.05298} = 2.23.$$

The p-value is

$$2 \times P(X \geq 2.23) = 0.028$$

where the random variable X has a t distribution with $125 - 1 = 124$ degrees of freedom.

There is some evidence that the manufacturing process needs adjusting but it is not overwhelming.

8.2.19 Consider the hypothesis testing problem

$$H_0 : \mu = 0.2 \quad \text{versus} \quad H_A : \mu \neq 0.2.$$

The test statistic is

$$t = \frac{\sqrt{n}(\bar{x} - \mu_0)}{s} = \frac{\sqrt{75} \times (0.23181 - 0.22500)}{0.07016} = 0.841.$$

The p-value is

$$2 \times P(X \geq 0.841) = 0.40$$

where the random variable X has a t distribution with $75-1 = 74$ degrees of freedom.

There is not sufficient evidence to conclude that the spray painting machine is not performing properly.

1.8 Supplementary Problems

1.8.1 $\mathcal{S} = \{1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6)$

1.8.2 If the four contestants are labeled A, B, C, D and the notation (X,Y) is used to indicate that contestant X is the winner and contestant Y is the runner up, then the sample space is $\mathcal{S} = \{(A,B), (A,C), (A,D), (B,A), (B,C), (B,D), (C,A), (C,B), (C,D), (D,A), (D,B), (D,C)\}$.

1.8.3 One way is to have the two team captains each toss the coin once. If one obtains a head and the other a tail, then the one with the head wins (this could just as well be done the other way around so that the one with the tail wins, as long as it is decided beforehand). If both captains obtain the same result, that is if there are two heads or two tails, then the procedure is repeated.

1.8.4 See Figure 1.10.
There are 36 equally likely outcomes, 16 of which have scores differing by no more than one.
Thus, P(scores on two dice differ by no more than one) $= \frac{16}{36} = \frac{4}{9}$.

1.8.5 Number of ways to pick a card = 52.
Number of ways to pick a diamond picture card = 3.
Thus, P(picking a diamond picture card) $= \frac{3}{52}$.

1.8.6 With replacement, P(drawing two hearts) $= \frac{13}{52} \times \frac{13}{52} = \frac{1}{16} = 0.0625$.

Without replacement, P(drawing two hearts) $= \frac{13}{52} \times \frac{12}{51} = \frac{3}{51} = 0.0588$.

The probability decreases without replacement.

1.8.7 $A = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1)\}$.
$B = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$.

(a) $A \cap B = \{(1,1), (2,2)\}$ and $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$.

(b) $A \cup B = \{(1,1), (1,2), (1,3), (2,1), (2,2), (3,1), (3,3), (4,4), (5,5), (6,6)\}$ and $P(A \cup B) = \frac{10}{36} = \frac{5}{18}$.

(c) $A' \cup B = \{(1,1), (1,4), (1,5), (1,6), (2,2), (2,3), (2,4), (2,5), (2,6), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$ and $P(A^1 \cup B) = \frac{32}{36} = \frac{8}{9}$.

1.8.8 See Figure 1.10. Let the notation (x, y) indicate that the score on the red die is x and the score on the blue die is y.

(a) The event *the sum of the scores on the two dice is eight* consists of the outcomes {(2,6), (3,5), (4,4), (5,3), (6,2)}.

Thus $P(\text{red die is 5} \mid \text{sum of scores is 8}) = \frac{P(\text{red die is 5} \cap \text{sum of scores is 8})}{P(\text{sum of scores is 8})}$

$= \frac{\left(\frac{1}{36}\right)}{\left(\frac{5}{36}\right)} = \frac{1}{5}$.

(b) $P(\text{either score is 5} \mid \text{sum of scores is 8}) = 2 \times \frac{1}{5} = \frac{2}{5}$.

(c) The event *the score on either die is 5* consists of the 11 outcomes {(1,5), (2,5), (3,5), (4,5), (5,5), (6,5), (5,6), (5,4), (5,3), (5,2), (5,1)}.

Thus $P(\text{sum of scores is 8} \mid \text{either score is 5}) = \frac{P(\text{sum of scores is 8} \cap \text{either score is 5})}{P(\text{either score is 5})}$

$= \frac{\left(\frac{2}{36}\right)}{\left(\frac{11}{36}\right)} = \frac{2}{11}$.

1.8.9 $P(A) = P(\text{either switch 1 or 4 is open or both})$
$= 1 - P(\text{both switches 1 and 4 are closed}) = 1 - 0.15^2 = 0.9775$.

$P(B) = P(\text{either switch 2 or 5 is open or both})$
$= 1 - P(\text{both switches 2 and 5 are closed}) = 1 - 0.15^2 = 0.9775$.

$P(C) = P(\text{switches 1 and 2 are both open}) = 0.85^2 = 0.7225$

$P(D) = P(\text{switches 4 and 5 are both open}) = 0.85^2 = 0.7225$

If $E = C \cup D$ then $P(E) = 1 - (P(C') \times P(D')) = 1 - (1 - 0.85^2)^2 = 0.923$.

P(message gets through the network)
$= (P(\text{switch 3 is open}) \times P(A) \times P(B)) + (P(\text{switch 3 closed}) \times P(E))$
$= (0.85 \times (1 - 0.15^2)^2) + (0.15 \times (1 - (1 - 0.85^2)^2)) = 0.9506$.

1.8.10 The sample space for the experiment of two coin tosses consists of the equally likely outcomes {(H,H), (H,T), (T,H), (T,T)}. Three of these four outcomes contain at least one head so that $P(\text{at least one head in two coin tosses}) = \frac{3}{4}$.

The sample space for four tosses of a coin consists of $2^4 = 16$ equally likely outcomes of which the following 11 outcomes contain at least two heads
{(HHTT), (HTHT), (HTTH), (THHT), (THTH), (TTHH), (HHHT), (HHTH), (HTHH), (THHH), (HHHH)}.
Therefore $P(\text{at least two heads in four coin tosses}) = \frac{11}{16}$ which is smaller than the previous probability.

1.8.11 (a) P(blue ball) = (P(bag 1) x P(blue ball | bag 1))
+ (P(bag 2) x P(blue ball | bag 2)) + (P(bag 3) x P(blue ball | bag 3))
+ (P(bag 4) x P(blue ball | bag 4))
= (0.15 x $\frac{7}{16}$) + (0.2 x $\frac{8}{18}$) + (0.35 x $\frac{9}{19}$) + (0.3 x $\frac{7}{11}$) = 0.5112.

(b) P(bag 4 | green ball) = $\frac{\text{P(green ball} \cap \text{bag 4)}}{\text{P(green ball)}}$ = $\frac{\text{P(bag 4) x P(green ball | bag 4)}}{\text{P(greenball)}}$
= $\frac{0.3\times 0}{\text{P(green ball)}}$ = 0.

(c) P(bag 1 | blue ball) = $\frac{\text{P(bag 1) x P(blue ball | bag 1)}}{\text{P(blue ball)}}$ = $\frac{0.15\times\frac{7}{16}}{0.5112}$
= $\frac{0.0656}{0.5112}$ = 0.128.

1.8.12 (a) $\mathcal{S}$ = {1, 2, 3, 4, 5, 6, 10)
(b) P(10) = P(score on die is 5) x P(tails) = $\frac{1}{6}$ x $\frac{1}{2}$ = $\frac{1}{12}$.
(c) P(3) = P(score on die is 3) × P(heads) = $\frac{1}{6}$ x $\frac{1}{2}$ = $\frac{1}{12}$.
(d) P(6) = P(score on die is 6) + (P(score on die is 3) x P(tails))
= $\frac{1}{6}$ + ($\frac{1}{6}$ × $\frac{1}{2}$) = $\frac{1}{4}$.
(e) 0.

(f) P(score on die is odd | 6 is recorded) = $\frac{\text{P(score on die is odd} \cap \text{6 is recorded)}}{\text{P(6 is recorded)}}$
= $\frac{\text{P(score on die is 3) x P(tails)}}{\text{P(6 is recorded)}}$ = $\frac{(\frac{1}{12})}{(\frac{1}{4})}$ = $\frac{1}{3}$.

1.8.13 5^4 = 625
4^5 = 1,024
In this case $5^4 < 4^5$ and in general, $n_2^{n_1} < n_1^{n_2}$ when $3 \leq n_1 < n_2$.

1.8.14 $\frac{20!}{5!\times 5!\times 5!\times 5!} = 1.17 \times 10^{10}$.
$\frac{20!}{4!\times 4!\times 4!\times 4\times 4!} = 3.06 \times 10^{11}$.

1.8.15 $P(X=0) = \frac{1}{4}$, $P(X=1) = \frac{1}{2}$, $P(X=2) = \frac{1}{4}$.
$P(X=0|\text{white}) = \frac{1}{8}$, $P(X=1|\text{white}) = \frac{1}{2}$, $P(X=2|\text{white}) = \frac{3}{8}$.
$P(X=0|\text{black}) = \frac{1}{2}$, $P(X=1|\text{black}) = \frac{1}{2}$, $P(X=2|\text{black}) = 0$.

1.8.16 Let A be the event that the order is from a first time customer and let B be the event that the order is dispatched within one day. It is given that $P(A) = 0.28$, $P(B|A) = 0.75$ and $P(A' \cap B') = 0.30$.

Therefore $P(A' \cap B) = P(A') - P(A' \cap B') = (1 - 0.28) - 0.30 = 0.42$, $P(A \cap B) = P(A) \times P(B|A) = 0.28 \times 0.75 = 0.21$, $P(B) = P(A' \cap B) + P(A \cap B) = 0.42 + 0.21 =$ 0.63, and finally $P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0.21}{0.63} = \frac{1}{3}$.

1.8.17 It is given that $P(\text{Puccini}) = 0.26$, $P(\text{Verdi}) = 0.22$, $P(\text{other composer}) = 0.52$, $P(\text{female}|\text{Puccini}) = 0.59$, $P(\text{female}|\text{Verdi}) = 0.45$ and $P(\text{female}) = 0.62$.

(a) Since $P(\text{female}) = P(\text{Puccini}) \times P(\text{female}|\text{Puccini}) + P(\text{Verdi}) \times P(\text{female}|\text{Verdi}) + P(\text{other composer}) \times P(\text{female}|\text{other composer})$ it follows that
$0.62 = (0.26 \times 0.59) + (0.22 \times 0.45) + (0.52 \times P(\text{female}|\text{other composer}))$
so that $P(\text{female}|\text{other composer}) = 0.7069$.

(b) $P(\text{Puccini}|\text{male}) = \frac{P(\text{Puccini})P(\text{male}|\text{Puccini})}{P(\text{male})} = \frac{0.26\times(1-0.59)}{1-0.62} = 0.281$.

1.8.18 The total number of possible samples is C^{92}_{10}.

(a) The number of samples that don't contain any fibers of polymer B is C^{75}_{10}. The answer is $\frac{C^{75}_{10}}{C^{92}_{10}} = \frac{75}{92} \times \frac{74}{91} \ldots \times \frac{66}{83} = 0.115$.

(b) The number of samples that contain exactly one fiber of polymer B is $17 \times C^{75}_{9}$. The answer is $\frac{17\times C^{75}_{9}}{C^{92}_{10}} = 0.296$.

(c) The number of samples that contain three fibers of polymer **A,** three fibers of polymer B and four fibers of polymer C is $C^{43}_{3} \times C^{17}_{3} \times C^{32}_{4}$. The answer is $\frac{C^{43}_{3}\times C^{17}_{3}\times C^{32}_{4}}{C^{92}_{10}} = 0.042$.

1.8.19 The total number of possible sequences of heads and tails is $2^5 = 32$, each being equally likely, and sixteen of which don't include a sequence of three outcomes of the same kind. Therefore the probability is $\frac{16}{32} = 0.5$.

1.8.20 (a) Calls answered by an experienced operator that last over five minutes.

(b) Successfully handled calls that were answered either within ten seconds or by an inexperienced operator (or both).

(c) Calls answered after ten seconds that lasted more than five minutes and that weren't handled successfully.

(d) Calls that were either answered within ten seconds and lasted less than five minutes, or that were answered by an experienced operator and were handled successfully.

1.8.21 (a) $\frac{20!}{7!\times 7!\times 6!} = 133{,}024{,}320$.

(b) If the first and the second job are assigned to production line I, the number of assignments is $\frac{18!}{5!\times 7!\times 6!} = 14{,}702{,}688$. If the first and the second job are assigned to production line II, the number of assignments is $\frac{18!}{7!\times 5!\times 6!} = 14{,}702{,}688$. If

the first and the second job are assigned to production line III, the number of assignments is ——— $-$ 10,501,920. The answer is $14,702,688 + 14,702,688 +$ 10,501,920 $=$ 39,907,296.

(c) The answer is 133,024,320 $-$ 39,907,296 $= 93,117,02$

2.7 Supplementary Problems

2.7.1 (a)

x_i	2	3	4	5	6
p_i	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

(b)
$$E(X) = \left(2\times\frac{2}{30}\right)+\left(3\times\frac{4}{30}\right)+\left(4\times\frac{6}{30}\right)+\left(5\times\frac{8}{30}\right)+\left(6\times\frac{10}{30}\right)$$
$$= \frac{14}{3}.$$

2.7.2 (a)

x_i	0	1	2	3	4	5	6
$F(x_i)$	0.21	0.60	0.78	0.94	0.97	0.99	1.00

(b)
$$E(X) = (0\times 0.21) + (1\times 0.39) + (2\times 0.18) + (3\times 0.16)$$
$$+ (4\times 0.03) + (5\times 0.02) + (6\times 0.01) = 1.51.$$

(c)
$$E(X^2) = (0^2\times 0.21) + (1^2\times 0.39) + (2^2\times 0.18) + (3^2\times 0.16)$$
$$+ (4^2\times 0.03) + (5^2\times 0.02) + (6^2\times 0.01) = 3.89.$$
$$Var(X) = 3.89 - (1.51)^2 = 1.61.$$

(d) The expectation is **1.51** x 60 = 90.6 and the variance is 1.61 x ou = 96.6.

2.7.3 (a)

x_i	2	3	4	5
p_i	$\frac{2}{30}$	$\frac{13}{30}$	$\frac{13}{30}$	$\frac{2}{30}$

(b)
$$E(X) = \left(2\times\frac{2}{30}\right)+\left(3\times\frac{13}{30}\right)+\left(4\times\frac{13}{30}\right)+\left(5\times\frac{2}{30}\right) = \frac{7}{2}.$$
$$E(X^2) = \left(2^2\times\frac{2}{30}\right)+\left(3^2\times\frac{13}{30}\right)+\left(4^2\times\frac{13}{30}\right)+\left(5^2\times\frac{2}{30}\right) = \frac{383}{30}.$$
$$Var(X) = E(X^2) - E(X)^2 = \frac{383}{30} - \left(\frac{7}{2}\right)^2 = \frac{31}{60}.$$

(c)

x_i	2	3	4	5
p_i	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{3}{10}$	$\frac{2}{10}$

$$E(X) = \left(2\times\frac{2}{10}\right)+\left(3\times\frac{3}{10}\right)+\left(4\times\frac{3}{10}\right)+\left(5\times\frac{2}{10}\right) = \frac{7}{2}.$$

$$E(X^2) = \left(2^2 \times \frac{2}{10}\right) + \left(3^2 \times \frac{3}{10}\right) + \left(4^2 \times \frac{3}{10}\right) + \left(5^2 \times \frac{2}{10}\right) = \frac{133}{10}.$$

$$Var(X) = E(X^2) - E(X)^2 = \frac{133}{10} - \left(\frac{7}{2}\right)^2 = \frac{21}{20}.$$

2.7.4 Let X_i be the value of the i^{th} card dealt. Then

$$E(X_i) = \left(2 \times \frac{1}{13}\right) + \left(3 \times \frac{1}{13}\right) + \left(4 \times \frac{1}{13}\right) + \left(5 \times \frac{1}{13}\right) + \left(6 \times \frac{1}{13}\right)$$

$$+ \left(7 \times \frac{1}{13}\right) + \left(8 \times \frac{1}{13}\right) + \left(9 \times \frac{1}{13}\right) + \left(10 \times \frac{1}{13}\right) + \left(15 \times \frac{4}{13}\right) = \frac{114}{13}.$$

The total score of the hand is $Y = X_1 + \ldots + X_{13}$ which has an expectation

$$E(Y) = E(X_1) + \ldots + E(X_{13}) = 13 \times \frac{114}{13} = 114.$$

2.7.5 (a) $\int_1^{11} A\left(\frac{3}{2}\right)^x dx = 1$ $\qquad A = \frac{\ln(1.5)}{1.5^{11} - 1.5} = \frac{1}{209.6}$

(b) $F(x) = \int_1^x \frac{1}{209.6}\left(\frac{3}{2}\right)^y dy = 0.01177\left(\frac{3}{2}\right)^x - 0.01765$
for $1 \leq x \leq 11$.

(c) Solving $F(x) = 0.5$ gives $x = 9.332$.

(d) Solving $F(x) = 0.25$ gives $x = 7.706$.
Solving $F(x) = 0.75$ gives $x = 10.305$.
The interquartile range is $10.30 - 7.71 = 2.599$.

2.7.6 (a) $f_X(x) = \int_1^2 4x(2-y)\, dy = 2x$ for $0 \leq x \leq 1$.

(b) $f_Y(y) = \int_0^1 4x(2-y)\, dx = 2(2-y)$ for $1 \leq y \leq 2$.

Since $f(x, y) = f_X(x) \times f_Y(y)$ the random variables **are** independent.

(c) $Cov(X, Y) = 0$ since the random variables are independent.

(d) $f_{X|Y=1.5}(x) = f_X(x)$ since the random variables are independent.

2.7.7 (a) $$\int_5^{10} A\left(x+\frac{2}{x}\right)\,dx = 1 \quad\Rightarrow\quad A = 0.02572.$$

(b) $$F(x) = \int_5^{x} 0.02572\left(y+\frac{2}{y}\right)\,dy = 0.0129x^2 + 0.0514\ln(x) - 0.404$$
for $5 \le x \le 10$.

(c) $$E(X) = \int_5^{10} 0.02572\,x\left(x+\frac{2}{x}\right) = 7.759.$$

(d) $$E(X^2) = \int_5^{10} 0.02572\,x^2\left(x+\frac{2}{x}\right) = 62.21.$$

$$Var(X) = E(X^2) - E(X)^2 = 62.21 - 7.759^2 = 2.01.$$

(e) Solving $F(x) = 0.5$ gives $x = 7.88$.

(f) Solving $F(x) = 0.25$ gives $x = 6.58$.
Solving $F(x) = 0.75$ gives $x = 9.00$.
The interquartile range is $9.00 - 6.58 = 2.42$.

(g) The expectation is $E(X) = 7.759$.
The variance is $Var(X)/10 = 0.0201$.

2.7.8 $$Var(a_1X_1 + a_2X_2 + \ldots + a_nX_n + b)$$
$$= Var(a_1X_1) + \ldots + Var(a_nX_n) + Var(b)$$
$$= a_1^2Var(X_1) + \ldots + a_n^2Var(X_n) + 0.$$

2.7.9 $Y = \frac{5}{3}X - 25$.

2.7.10 Notice that $E(Y) = aE(X) + b$ and $Var(Y) = a^2Var(X)$. Also

$$Cov(X,Y) = E(\,(X - E(X))\,(Y - E(Y))\,) = E(\,(X - E(X))\,a(X - E(X))\,)$$
$$= aVar(X).$$

Therefore

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sqrt{Var(X)Var(Y)}} = \frac{aVar(X)}{\sqrt{Var(X)a^2Var(X)}} = \frac{a}{|a|}$$

which is 1 if $a > 0$ and -1 if $a < 0$.

2.7.11 The expected amount of a claim is

$$E(X) = \int_0^{1,800} x \frac{x(1,800-x)}{972,000,000} dx = \$900.$$

Consequently the expected profit from each customer is \$100 − \$5 − (0.1 x \$900) = \$5. The expected profit from 10,000 customers is 10,000 × \$5 = \$50,000.

The profits may or may not be independent depending on the type of insurance and pool of customers.. If large natural disasters affect the customers then the claims would not be independent.

2.7.12 (a) The expectation is 5 x 320 = 1600 seconds.
The variance is $5 \times 63^2 = 19,845$ and the standard deviation is $\sqrt{19,845} = 140.9$ seconds.

(b) The expectation is 320 seconds.
The variance is $63^2/10 \doteq 396.9$ and the standard deviation is $\sqrt{396.9} = 19.92$ seconds.

2.7.13 (a) The state space is the positive integers from 1 to n, with each outcome having a probability value of $1/n$.

(b) $E(X) = \left(\frac{1}{n} \times 1\right) + \left(\frac{1}{n} \times 2\right) + \ldots + \left(\frac{1}{n} \times n\right) = \frac{n+1}{2}.$

(c) $E(X^2) = \left(\frac{1}{n} \times 1^2\right) + \left(\frac{1}{n} \times 2^2\right) + \ldots + \left(\frac{1}{n} \times n^2\right) = \frac{(n+1)(2n+1)}{6}$
Therefore $Var(X) = E(X^2) - (E(X))^2 = \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2 = \frac{n^2-1}{12}.$

2.7.14 (a) In this case $X = X_1 + X_2$, the sum of the times of the two rides. The expectation is $E(X) = E(X_1) + E(X_2) = 87 + 87 = 174$ minutes. The variance is $Var(X) = Var(X_1) + Var(X_2) = 3^2 + 3^2 = 18$, and the standard deviation is $\sqrt{18} = 4.24$ minutes.

(b) In this case $X = 2 \times X_1$, twice the time of the ride. The expectation is $E(X) = 2 \times E(X_1) = 2 \times 87 = 174$ minutes. The variance is $Var(X) = 2^2 \times Var(X_1) = 2^2 \times 3^2 = 36$, and the standard deviation is $\sqrt{36} = 6$ minutes.

3.6 Supplementary Problems

3.6.1 (a) $P(B(18, 0.085) \geq 3) = 1 - P(B(18, 0.085) \leq 2) = 0.1931$.

(b). $P(B(18, 0.085) \leq 1) = 0.5401$.

(c) $18 \times 0.085 = 1.53$.

3.6.2 $P(B(13, 0.4) \geq 3) = 1 - P(B(13, 0.4) \leq 2) = 0.9421$.

The expected number of cells is $13 + (13 \times 0.4) = 18.2$.

3.6.3 (a)

$$\frac{8!}{2! \times 3! \times 3!} \times 0.40^2 \times 0.25^3 \times 0.35^3 = 0.0600.$$

(b)

$$\frac{8!}{3! \times 1! \times 4!} \times 0.40^3 \times 0.25' \times 0.35' = 0.0672.$$

(c) $P(B(8, 0.35) \leq 2) = 0.4278$.

3.6.4 (a)

$$P(X = 0) = \frac{e^{-2/3} \times (2/3)^0}{0!} = 0.5134.$$

(b)

$$P(X = 1) = \frac{e^{-2/3} \times (2/3)^1}{1!} = 0.3423;$$

(c) $P(X \geq 3) = 1 - P(X \leq 2) = 0.0302$.

3.6.5

$$P(X = 2) = \frac{e^{-3.3} \times (3.3)^2}{2!} = 0.2008.$$

$P(X \geq 6) = 1 - P(X \leq 5) = 0.1171$.

3.6.6 (a) Consider a negative binomial distribution with parameters $p = 0.55$ and $r = 4$.

(b) $P(X = 7) = \binom{6}{3} \times (1 - 0.55)^3 \times 0.55^4 = 0.1668$.

(c) $P(X = 6) = \binom{5}{3} \times (1 - 0.55)^2 \times 0.55' = 0{,}1853$.

(d) The probability that team A **wins** the series in game 5 is

$\binom{4}{3} \times (1 - 0.55)^1 \times 0.55^4 = 0.1647$.

The probability that team B **wins** the series in game 5 is

$\binom{4}{3} \times (1 - 0.45)^1 \times 0.45^4 = 0.0902$.

The probability that the series is over after game five is $0.1647 + 0.0902 = 0.2549$.

(e) The probability that team **A** wins the series in game 4 is $0.55^4 = 0.0915$. The probability that team **A** wins the series is $0.0915 + 0.1647 + 0.1853 + 0.1668 = 0.6083$.

3.6.7 (a) Consider a negative binomial distribution with parameters $p = 0.58$ and $r = 3$.

$$P(X = 9) = \binom{8}{2} \times (1 - 0.58)^6 \times 0.58^3 = 0.0300$$

(b) Consider a negative binomial distribution with parameters $p = 0.42$ and $r = 4$.
$P(X \leq 7) = P(X = 4) + P(X = 5) + P(X = 6) + P(X = 7) = 0.3294.$

3.6.8

$$P(\textit{two red balls}|\textit{head}) = \frac{\binom{6}{2} \times \binom{5}{1}}{\binom{11}{3}} = \frac{5}{11}.$$

$$P(\textit{two red balls}|\textit{tail}) = \frac{\binom{5}{2} \times \binom{6}{1}}{\binom{11}{3}} = \frac{4}{11}.$$

Then

$$P(\textit{two red balls}) = (P(\textit{head}) \times P(\textit{two red balls}|\textit{head})) + (P(\textit{tail}) \times P(\textit{two red balls}|\textit{tail}))$$

$$= \left(0.5 \times \frac{5}{11}\right) + \left(0.5 \times \frac{4}{11}\right) = \frac{9}{22}$$

and

$$P(\textit{head}|\textit{two red balls}) = \frac{P(\textit{head and two red balls})}{P(\textit{two red balls})}$$

$$= \frac{P(\textit{head}) \times P(\textit{two red balls}|\textit{head})}{P(\textit{two red balls})} = \frac{5}{9}.$$

3.6.9 Using the hypergeometric distribution, the answer is

$$P(X = 0) + P(X = 1) = \frac{\binom{36}{5} \times \binom{4}{0}}{\binom{40}{5}} + \frac{\binom{36}{4} \times \binom{4}{1}}{\binom{40}{5}} = 0.9310.$$

For a collection of 4,000,000 items of which 400,000 are defective, a $B(5, 0.1)$ distribution ***can*** be used.

$$P(X=0)+P(X=1)=\binom{5}{0}\times 0.1^0\times 0.9^5+\binom{5}{1}\times 0.1^1\times 0.9^4=0.9185$$

4.6 Supplementary Problems

4.6.1 $F(0) = P(winnings = 0) = \frac{1}{4}$,

$F(x) = P(winnings \leq x) = \frac{1}{4} + \frac{x}{720}$ for $0 \leq x \leq 360$,

$F(x) = P(winnings \leq x) = \frac{\sqrt{x+72{,}540}}{360}$ for $360 \leq x \leq 57{,}060$,

$F(x) = 1$ for $57{,}060 \leq x$.

4.6.2 (a) $\frac{0.693}{\lambda} = 1.5 \Rightarrow \lambda = 0.462$.

(b) $P(X \geq 2) = 1 - F(2) = 1 - (1 - e^{-0.462\times 2}) = e^{-0.924} = 0.397$.

$P(X \leq 1) = F(1) = 1 - e^{-0.462\times 1} = 0.370$.

4.6.3 (a) $E(X) = \frac{1}{0.7} = 1.4286$.

(b) $P(X \geq 3) = 1 - F(3) = 1 - (1 - e^{-0.7\times 3}) = e^{-2.1} = 0.1225$.

(c) $\frac{0.693}{0.7} = 0.9902$.

(d) A Poisson distribution with parameter $0.7 \times 10 = 7$.

(e) $P(X \geq 5) = 1 - P(X = 0) - P(X = 1) - P(X = 2) - P(X = 3) - P(X = 4)$ $= 0.8270$.

(f) A gamma distribution with parameters $k = 10$ and $\lambda = 0.7$.

$E(X) = \frac{10}{0.7} = 14.286$. $Var(X) = \frac{10}{0.7^2} = 20.408$.

4.6.4 (a) $E(X) = \frac{1}{5.2} = 0.1923$.

(b) $P(X \leq 1/6) = F(1/6) = 1 - e^{-5.2\times 1/6} = 0.5796$.

(c) A gamma distribution with parameters $k = 10$ and $\lambda = 5.2$.

(d) $E(X) = \frac{10}{5.2} = 1.923$.

(e) $P(X > 5) = 0.4191$ where the random variable X has a Poisson distribution with parameter 5.2.

4.6.5 (a) The total area under the triangle is one so the height at the midpoint is $\frac{2}{b-a}$.

5.5 Supplementary Problems

5.5.1 (a) $P(N(500, 50^2) \geq 625) = 0.0062$.

(b) Solving $P(N(500, 50^2) \leq x) = 0.99$ gives $x = 616.3$.

(c) $P(N(500, 50^2) \geq 700) \simeq 0$.
Strong suggestion that an eruption is imminent.

5.5.2 (a) $P(N(12500, 200000) \geq 13{,}000) = 0.1318$.

(b) $P(N(12500, 200000) \leq 11{,}400) = 0.0070$.

(c) $P(12{,}200 \leq N(12500, 200000) \leq 14{,}000) = 0.7484$.

(d) Solving $P(N(12500, 200000) \leq x) = 0.95$ gives $x = 13{,}200$.

5.5.3 (a) $P(N(70, 5.4^2) \geq 80) = 0.0320$.

(b) $P(N(70, 5.4^2) \leq 55) = 0.0027$.

(c) $P(65 \leq N(70, 5.4^2) \leq 78) = 0.7536$.

(d) Need $c = \sigma \times z_{0.025} = 5.4 \times 1.9600 = 10.584$.

5.5.4 (a) $P(X_1 - X_2 \geq 0) = P(N(0, 2 \times 5.4^2) \geq 0) = 0.5$.

(b) $P(X_1 - X_2 \geq 10) = P(N(0, 2 \times 5.4^2) \geq 10) = 0.0952$.

(c) $P(\frac{X_1+X_2}{2} - X_3 \geq 10) = P(N(0, 1.5 \times 5.4^2) \geq 10) = 0.0653$.

5.5.5 $P(|X_1 - X_2| \leq 3) = P(|N(0, 2 \times 2^2)| \leq 3) = P(-3 \leq N(0, 8) \leq 3) = 0.7112$.

5.5.6 $E(X) = \frac{1.43+1.60}{2} = 1.515$.
$Var(X) = \frac{(1.60-1.43)^2}{12} = 0.002408$.
The required probability can be estimated as
$P(180 \leq N(120 \times 1.515, 120 \times 0.002408) \leq 182) = 0.6447$.

6.5 Supplementary Problems

6.5.1 The population from which the sample is drawn should be all the birds on the island. However, the sample may not be representative if some species are more likely to be observed than others.

It appears that the grey markings are the most common followed by the black markings.

6.5.2 There do not appear to be any seasonal effects although there may possibly be a correlation from one month to the next.

The sample mean is $\bar{x} = 17.79$.

The sample median is 17.

The sample trimmed mean is 17.36.

The sample standard deviation is $s = 6.16$.

The upper sample quartile is 21.75.

The lower sample quartile is 14.

6.5.3 One question of interest in interpreting the data set is whether the month of sampling is representative of other months.

The sample is skewed.

The most frequent data value (the sample mode) is one error.

The sample mean is $\bar{x} = 1.633$.

The sample median is 1.5.

The sample trimmed mean is 1.615.

The sample standard deviation is $s = 0.999$.

The upper samplequartile is 2.

The lower sample quartile is 1.

6.5.4 The population could be all adult males who visit the clinic. This could be representative of all adult males in the population unless there is something special about the clientele of the clinic.

The largest observation 75.9 looks like an outlier on a histogram but may be a valid observation.

The sample mean is $\bar{x} = 69.618$.

The sample median is 69.5.

The sample trimmed mean is 69.513.

The sample standard deviation is $s = 1.523$.

The upper sample quartile is 70.275.

7.5 Supplementary Problems

7.5.1 $\text{bias}(\hat{\mu}_1) = 5 - \frac{\mu}{2}$.

$\text{bias}(\hat{\mu}_2) = 0$.

$Var(\hat{\mu}_1) = \frac{1}{8}$.

$Var(\hat{\mu}_2) = \frac{1}{2}$.

$MSE(\hat{\mu}_1) = \frac{1}{8} + (5 - \frac{\mu}{2})^2$.

$MSE(\hat{\mu}_2) = \frac{1}{2}$.

7.5.2 (a) $\text{bias}(\hat{p}) = -\frac{p}{7}$.

(b) $Var(\hat{p}) = \frac{3p(1-p)}{49}$.

(c) $MSE(\hat{p}) = \quad + (\frac{p}{7})^2 = \frac{3p-2p^2}{49}$.

(d) $MSE(X/12) = \frac{p(1-p)}{12}$.

7.5.3 (a) $F(t) = P(T \leq t) = P(X_1 \leq t) \times \ldots \times P(X_n \leq t)$
$= \frac{t}{\theta} \times \ldots \times \frac{t}{\theta} = (\frac{t}{\theta})^n$ for $0 \leq t \leq \theta$.

(b) $f(t) = \frac{dF(t)}{dt} = n\,\frac{t^{n-1}}{\theta^n}$ for $0 \leq t \leq \theta$.

(c) Notice that

$$E(T) = \int_0^\theta t\, f(t)\, dt = \frac{n}{n+1}\theta,$$

so that $E(\hat{\theta}) = 0$.

(d) Notice that

$$E(T^2) = \int_0^\theta t^2\, f(t)\, dt = \frac{n}{n+2}\theta^2,$$

so that

$$Var(T) = \frac{n}{n+2}\theta^2 - \left(\frac{n}{n+1}\theta\right)^2 = \frac{n\theta^2}{(n+2)(n+1)^2}.$$

Consequently,

$$Var(\hat{\theta}) = \frac{(n+1)^2}{n^2}Var(T) = \frac{\theta^2}{n(n+2)}$$

and

$$s.e.(\hat{\theta}) = \frac{\hat{\theta}}{\sqrt{n(n+2)}}.$$

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(e) $\hat{\theta} = \frac{11}{10} \times 7.3 = 8.03.$
$s.e.(\hat{\theta}) = \frac{8.03}{\sqrt{10 \times 12}} = 0.733.$

7.5.4 Recall that $f(x_i, \theta) = \frac{1}{\theta}$ for $0 \leq x_i \leq \theta$ (and $f(x_i, \theta) = 0$ elsewhere) so that the likelihood is $\frac{1}{\theta^n}$ as long as $x_i \leq \theta$ for $1 \leq i \leq n$ and is equal to zero otherwise.
$\text{bias}(\theta) = -\frac{\theta}{n+1}.$

7.5.5 Using the method of moments

$$E(X) = \frac{1}{p} = \bar{x}$$

which gives $\hat{p} = \frac{1}{\bar{x}}$.
The likelihood is

$$L(x_1, \ldots, x_n, \lambda) = p^n (1-p)^{x_1 + \ldots + x_n - n}$$

which is maximimized at $\hat{p} = \frac{1}{\bar{x}}$.

7.5.6 $\hat{p} = \frac{35}{100} = 0.35.$

$$s.e.(\hat{p}) = \sqrt{\frac{\hat{p}\,(1-\hat{p})}{n}} = \sqrt{\frac{0.35 \times 0.65}{100}} = 0.0477.$$

7.5.7 $\hat{\mu} = \bar{x} = 17.79.$

$$s.e.(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{6.16}{\sqrt{24}} = 1.26.$$

7.5.8 $\hat{\mu} = \bar{x} = 1.633.$

$$s.e.(\bar{x}) = \frac{s}{\sqrt{n}} = \frac{0.999}{\sqrt{30}} = 0.182.$$